Structure of Nonequilibrium Boundary Layer along a Flat Plate in a Partially Ionized Gas

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Theme

THIS paper presents the calculated results for the profiles of electron temperature and electron-ion pair concentration in a nonequilibrium, partially ionized boundary layer which is developed along a flat plate at floating potential. The boundary conditions at the wall for electron temperature and concentration of electron-ion pair are determined from the conditions for the continuities of electron energy flux and ion mass flux at the outer edge of a sheath, and the zero net current at the wall. Calculations have been carried out for a chemically frozen flow of an argon gas.

Content

In the case of a partially ionized gas, a sheath formed next to the wall is thin compared to the boundary-layer thickness, so that an ionized gas in the boundary layer is electrically neutral. Therefore, since ions move with electrons, the concept of ambipolar diffusion can be adopted.

If a flat plate is electrically insulated from ground or if electric field is not applied to it, it will be at floating potential. Since the plate potential is generally lower than the plasma potential, an ion sheath is formed, so that only electrons which overcome the potential difference between the wall and the plasma can reach the wall in addition to ions. However, the net current normal to the wall must be zero. The boundary layer of a partially ionized gas is governed only by the transport properties of neutral atoms, and neutral atoms are essentially uninfluenced by the presence of electrons.

In accord with the aforementioned boundary-layer model, we make the following assumptions: 1) partially ionized argon, 2) no external electric or magnetic field, 3) all species have the same mass motion velocity, 4) $T_a = T_i$, 5) steady flow, 6) freestream conditions constant along x, 7) collision-free plasma sheath, 8) chemically frozen flow, 9) ambipolar diffusion, $n_e = n_i$, $V_{d_e} = V_{d_i}$.

In application to a real flow situation, it is more valid to consider recombination process in a boundary layer. In this paper, to investigate the dependences of electron-temperature profile on the wall temperature and on the location from leading edge, the flow model is simplified and chemically frozen flow is assumed. This means that the amount of energy transferred to the electron gas due to recombination is assumed negligible, and a thermally equilibrium state in external flow is assumed in the present paper.

The governing equations for a chemically frozen boundary layer along a flat plate are written as follows[1]:

$$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y = 0 \tag{1}$$

$$\rho u(\partial u/\partial x) + \rho v(\partial u/\partial y) = -dp/dx + (\partial/\partial y)[\mu(\partial u/\partial y)] \tag{2}$$

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y}\left[\frac{\mu}{Pr}\frac{\partial H}{\partial y} + \mu\left(1 - \frac{1}{Pr}\right) \times \frac{\partial}{\partial y}\left(\frac{u^2}{2}\right) + \rho D_a\left(1 - \frac{1}{Le}\right)\left(\frac{I}{m_a} + c_p T_e\right)\frac{\partial c}{\partial y} + (\lambda_e - \lambda)\frac{\partial T_e}{\partial y}\right] \tag{3}$$

$$\rho u(\partial c/\partial x) + \rho v(\partial c/\partial y) = (\partial/\partial y)[\rho D_a(\partial c/\partial y)] \tag{4}$$

$$\frac{\partial}{\partial x}\left(\frac{3}{2} n_e k u T_e\right) + \frac{\partial}{\partial y}\left(\frac{3}{2} n_e k v T_e\right) = \frac{\partial}{\partial y}\left(\lambda_e \frac{\partial T_e}{\partial y} - \rho c_e V_{d_e} h_e\right) - n_e k T_e\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) + R \tag{5}$$

where standard notations are used, and especially, D_a = ambipolar diffusion coefficient, $c = (\rho_i + \rho_e)/\rho$, $c_e = \rho_e/\rho$, V_{de} = electron diffusion velocity equal to ambipolar diffusion velocity, I = ionization energy and subscripts e, i = electron, ion, respectively. The thermal conductivity λ and energy transferred due to elastic collision R were given by Jaffrin[2] and Appleton et al.,[3] respectively.

Boundary conditions are

$$u(\infty) = u_\infty, H(\infty) = H_\infty, c(\infty) = c_\infty, T_e(\infty) = T_{e_\infty} \tag{6}$$

$$u(0) = v(0) = 0, H(0) = H_w$$

Boundary conditions at the wall for electron temperature and concentration of electron-ion pairs are determined as follows. For the present problem, in which the wall is at floating potential, the first relation can be obtained from Langmuir-probe theory if the electrons are Maxwellian

$$(n_{e_s}\langle V_e\rangle/4)\exp(-e\Delta\phi/kT_{e_s}) - n_{i_s} e V_i = 0 \tag{7}$$

where $\langle V_e\rangle = (8kT_{e_s}/\pi m_e)^{1/2}$, $V_i = (kT_{e_s}/m_i)^{1/2}$, e = electronic charge, $\Delta\phi$ = potential difference between wall and plasma and subscript s = sheath edge. The next relation is obtained from the continuity of mass flow of ions at the outer

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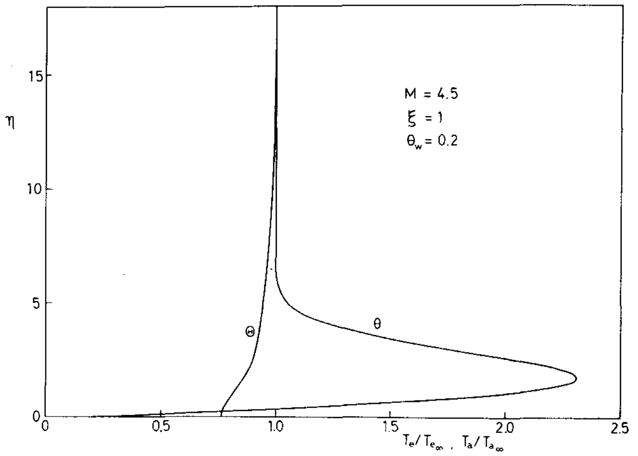

Fig. 1 Comparison of profiles of electron temperature and atom-ion temperature.

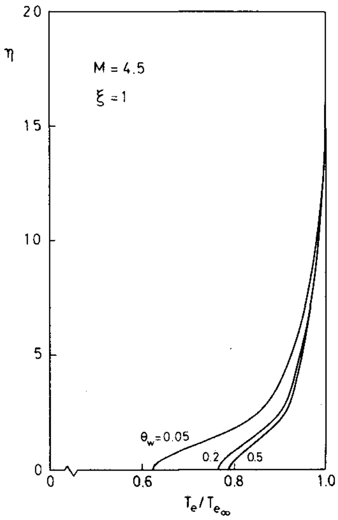

Fig. 2 Electron temperature profiles for various values of θ_w.

edge of the sheath

$$\rho_s D_a(\partial c/\partial y)_s = \rho_s c_s V_i \tag{8}$$

A third relation is obtained from the continuity of electron energy flux through the outer edge of the sheath

$$[\lambda_e(\partial T_e/\partial y) - \rho c_e V_{d_e} h_e]_s = (2kT_{e_s} + e\Delta\phi)(n_{e_s}\langle V_e\rangle/4)\exp(-e\Delta\phi/kT_{e_s}) \tag{9}$$

Since the sheath is very thin compared to the boundary-layer thickness, these boundary conditions are taken to be those at the wall.

The transformation of coordinates

$$\xi(x) = x,\ \eta(x,y) = \left(\frac{u_\infty}{\rho_\infty\mu_\infty\xi}\right)^{1/2}\int_0^y \rho dy \tag{10}$$

and the dimensionless stream function $f(\xi,\eta) = \psi/(\rho_\infty\mu_\infty u_\infty\xi)^{1/2}$, where ψ is a stream function, are used so that $u = u_\infty(\partial f/\partial\eta)$. The new variables $g = H/H_\infty$, $z = c/c_\infty$, $\Theta = T_e/T_{e_\infty}$, and $\theta = T_a/T_{a_\infty}$ are introduced. By assuming local similarity, Eqs. (1–5) are transformed to

$$(lf'')' + \tfrac{1}{2}ff'' = 0 \tag{11}$$

$$\left(\frac{l}{Pr}g'\right)' + \frac{1}{2}fg' = \left[\frac{l}{Sc}\left(\frac{1}{Le} - 1\right)\left(\frac{1}{H_\infty}\frac{I}{m_a} + \frac{h_{a\infty}}{H_\infty}\tau\Theta\right)c_\infty z'\right]' + \frac{u_\infty^2}{H_\infty}\left[l\left(\frac{1}{Pr} - 1\right)f'f''\right]' + \left[\frac{l}{Pr}\tau\frac{h_{a_\infty}}{H_\infty}\left(c_\infty z - \frac{\lambda_e}{\lambda}\right)\Theta'\right]' \tag{12}$$

$$[(l/Sc)z']' + \tfrac{1}{2}fz' = 0 \tag{13}$$

$$\Theta'' + \left(\frac{5}{2}\frac{\Theta'}{\Theta} + \frac{\theta'}{\theta} + \frac{a}{2}\frac{z\theta f}{\Theta^{5/2}}\right)\Theta' + \frac{a}{3}\frac{f\theta}{\Theta^{5/2}}\left(\frac{3}{2}z' + z\frac{\theta'}{\theta}\right)\Theta + b\xi\frac{z^2}{\Theta^4}\left(\frac{\theta}{\tau} - \Theta\right) = 0 \tag{14}$$

where

$$a = [3(1 + 2^{1/2})/5]Pr c_\infty(\epsilon/\tau^{1/2})Q_{ee_\infty}/Q_{aa_\infty}$$

$$b = 256(2 + 2^{1/2})/25\pi(\mu_\infty/\rho_\infty u_\infty)(\epsilon Q_{ee_\infty} n_{i_\infty})^2$$

Boundary conditions are

$$f'(\infty) = 1,\ g(\infty) = 1,\ z(\infty) = 1,\ \Theta(\infty) = 1$$

$$f(0) = f'(0) = 0,\ g(0) = g_w,\ z'(0) = ScRe_\infty^{1/2}(V_i/u_\infty)(1/\theta_w)z(0)$$

$$\Theta'(0) = \frac{64(1 + 2^{1/2})}{75\pi^{1/2}}\left[-\frac{1}{2} - \frac{1}{2}\log_e(2\pi) - \log_e\epsilon\right] \times \left(\frac{\mu_\infty}{\rho_\infty u_\infty}\right)^{1/2}\epsilon n_{i_\infty}Q_{ee_\infty}\xi^{1/2}\frac{z_w}{\Theta_w}$$

where $\epsilon = (m_e/m_a)^{1/2}$, $\tau = T_{e\infty}/T_{a\infty}$ and $Re_\infty = \rho_\infty u_\infty x/\mu_\infty$.

The following conditions are chosen as freestream ones: $T_{a_\infty} = T_{e_\infty} = 1000°$K, $n_{a_\infty} = 5 \times 10^{15}$ 1/cm^3, $n_{i_\infty} = n_{e_\infty} = 5 \times 10^{12}$ 1/cm^3 and $M = 4.5$. Argon is treated as a model gas so that $\epsilon = 1/275$. *Pr*, *Sc*, and *Le* are taken to be unity in order to simplify the analysis. The calculations have been carried out with the Runge-Kutta-Gill method, using a digital computer HITAC 5020 (KDC-II).

Figure 1 shows the comparison between electron temperature and atom-ion temperature distributions in the boundary layer. It is clearly seen that the thermal-layer thickness for the electrons is much thicker than that of the atoms and ions and that the electron temperature at the wall is higher than the atom-ion temperature there. This result is convenient to the use of Eq. (7), since Eq. (7) may be used only in the case where $T_e > T_i$. Figure 2 shows the electron-temperature profiles in the boundary layer for various values of θ_w.

References

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